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COUPLED QUADRUPOLE AND MONOPOLE VIBRATIONS OF LARGE AMPLITUDE

E.B.Balbutsev, I.V.Molodtsova, P.Schuck*

The set of nonlinear dynamical equations for quadrupole and monopole moments of nuclei is derived from the equation for Wigner function f (**r**, **p**, t) with the help of the method of Wigner function moments. These equations are solved numerically for ^{208}Pb. The giant quadrupole and monopole resonances are reproduced very well. The corresponding multiphonon states are predicted.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Связанные квадрупольные и монопольные колебания большой амплитуды

Е.Б.Бальбуцев, И.В.Молодцова, П.Шук

Из уравнения для функции Вигнера f (**r**, **p**, t) с помощью метода моментов функции Вигнера выведена система нелинейных динамических уравнений для квадрупольного и монопольного моментов ядра. Полученные уравнения решены численно для ^{208}Pb. Хорошо воспроизводятся энергии гигантских квадрупольного и монопольного резонансов. Предсказаны соответствующие мультифононные резонансы.

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1. Introduction

Large amplitude motion is very interesting and complicated field of nuclear physics. There are not so much methods and models treating this problem ([1], [2], [3] and references therein), so every new result here is valuable.

In this paper we use the method of Wigner function moments in the frame of TDHF theory with the simple Hamiltonian to derive the set of nonlinear dynamical equations for the quadrupole and monopole moments of nucleus. This model is attractive, because it allows one to write exact equations, which can be solved exactly. And what is more, it can be generalized to become rather realistic.

*ISN, Grenoble, France

2. Equations of motion

2.1. *Description of the Model.* Our model is based on the equation for a one-body density matrix $\hat{\rho} = \rho(\mathbf{r}_1, \mathbf{r}_2, t)$ in the TDHF theory:

$$\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}] . \tag{1}$$

We transform it into the equation for Wigner function [4]

$$f(\mathbf{r}, \mathbf{p}, t) = \frac{1}{(2\pi h)^3}\int e^{-i\mathbf{p}\cdot\mathbf{s}/h}\rho(\mathbf{r}+\frac{\mathbf{s}}{2}, \mathbf{r}-\frac{\mathbf{s}}{2}, t)d\mathbf{s}:$$

$$\frac{\partial f}{\partial t} = \frac{2}{h}\sin\left\{\frac{h}{2}(\nabla_{\mathbf{r}}^{H}\cdot\nabla_{\mathbf{p}}^{f} - \nabla_{\mathbf{p}}^{H}\cdot\nabla_{\mathbf{r}}^{f})\right\}H_W f , \tag{2}$$

where upper index of nabla shows the function which this operator acts on, H_W is the Wigner transform of the Hamiltonian:

$$H_W(\mathbf{r}, \mathbf{p}) = \int e^{-i\mathbf{p}\cdot\mathbf{s}/h}(\mathbf{r}+\frac{\mathbf{s}}{2}|H|\,\mathbf{r}-\frac{\mathbf{s}}{2})d\mathbf{s} .$$

If the Hamiltonian is the sum of the kinetic energy and the local potential $V(\mathbf{r})$, its Wigner transform is just the classical version of the same Hamiltonian: $H_W = \frac{p^2}{2m} + V(\mathbf{r})$. Then equation (2) becomes:

$$\frac{\partial f}{\partial t} + \frac{1}{m}\mathbf{p}\cdot\nabla_{\mathbf{r}} f = \frac{2}{h}\sin\left(\frac{h}{2}\nabla_{\mathbf{r}}^{V}\cdot\nabla_{\mathbf{p}}^{f}\right)Vf . \tag{3}$$

Our model potential is the harmonic oscillator with the quadrupole-quadrupole residual interaction: $V(\mathbf{r}, t) = \frac{1}{2}m\omega^2 r^2 + \lambda q(t)Q(\mathbf{r})$, where $Q(\mathbf{r}) = x_1^2 + x_2^2 - 2x_3^2$, $q(t) = trQ\rho =$ $= \int d\mathbf{r}d\mathbf{p}Q(\mathbf{r})f(\mathbf{r}, \mathbf{p}, t)$. Only the first term of the sin-operator survives in this case and we have:

$$\frac{\partial f}{\partial t} + \frac{1}{m}\sum_{i=1}^{3} p_i\frac{\partial f}{\partial x_i} - \sum_{i=1}^{3}\frac{\partial V}{\partial x_i}\frac{\partial f}{\partial p_i} = 0 , \tag{4}$$

with

$$\frac{\partial V}{\partial x_i} = [m\omega^2 + 2\lambda q(t)(\delta_{i1} + \delta_{i2} - 2\delta_{i3})]x_i . \tag{5}$$

2.2. *Wigner Function Moments.* Now we apply the method of Wigner function moments [5] to derive the closed system of the dynamical equations for cartesian tensors of a second rank. Integrating equation (4) over the phase space $\{\mathbf{r},\mathbf{p}\}$ with the weights $x_i x_j$, $p_i x_j$, $p_i p_j$ we get:

$$\int x_i x_j \frac{\partial n(\mathbf{r}, t)}{\partial t} d\mathbf{r} + \int x_i x_j \frac{\partial (n(\mathbf{r}, t) u_s(\mathbf{r}, t))}{\partial x_s} d\mathbf{r} = 0 , \tag{6}$$

$$m \int x_j \frac{\partial}{\partial t} (n(\mathbf{r}, t) u_i(\mathbf{r}, t)) d\mathbf{r} + \int n(\mathbf{r}, t) x_j \frac{\partial V}{\partial x_i} d\mathbf{r} + \frac{1}{m} \int x_j \frac{\partial}{\partial x_s} \mathcal{A}_{si}(\mathbf{r}, t) d\mathbf{r} = 0 , \tag{7}$$

$$\frac{\partial}{\partial t} \int \mathcal{A}_{ij}(\mathbf{r}, t) d\mathbf{r} + m \int n(\mathbf{r}, t) \left[u_i(\mathbf{r}, t) \frac{\partial V}{\partial x_j} \right]_{ij} d\mathbf{r} + \frac{1}{m} \int \frac{\partial}{\partial x_s} \mathcal{A}_{sij}(\mathbf{r}, t) d\mathbf{r} = 0 , \tag{8}$$

where $[...]_{ij}$ means that the quantity into brackets is symmetrized with respect to idexes i and $j([a_i b_j]_{ij} = a_i b_j + a_j b_i)$ and the summation over repeated indexes is assumed. Here we have introduced the notations: $n(\mathbf{r}, t) = \int f(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}$, $mn(\mathbf{r}, t) u_i(\mathbf{r}, t) = \int p_i f(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}$, $A_{ij...k}(\mathbf{r}, t) = \int p_i p_j ... p_k f(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}$. By definition $n(\mathbf{r}, t)$ is the nucleon density, $\mathbf{u}(\mathbf{r}, t)$ is the mean velocity of nucleons, $\mathcal{A}_{ij}(\mathbf{r}, t)/2m$ is the kinetic energy tensor (or pressure tensor). Integrating by parts the last terms in (6)—(8) and introducing the notations $J_{ij}(t) = \int x_i x_j n(\mathbf{r}, t) d\mathbf{r}$ for an inertia tensor and $\Pi_{ij}(t) = \int \mathcal{A}_{ij}(\mathbf{r}, t) d\mathbf{r}$ for an integral kinetic energy tensor we have:

$$\frac{d}{dt} J_{ij}(t) - [\int x_j n(\mathbf{r}, t) u_i(\mathbf{r}, t) d\mathbf{r}]_{ij} = 0 , \tag{9}$$

$$m \frac{d}{dt} \int x_j n(\mathbf{r}, t) u_i(\mathbf{r}, t) d\mathbf{r} + \int x_j n(\mathbf{r}, t) \frac{\partial V}{\partial x_i} d\mathbf{r} - \frac{1}{m} \Pi_{ij}(t) = 0, \tag{10}$$

$$\frac{d}{dt} \Pi_{ij}(t) + m \int n(\mathbf{r}, t) \left[u_i(\mathbf{r}, t) \frac{\partial V}{\partial x_j} \right]_{ij} d\mathbf{r} = 0. \tag{11}$$

The last integral of the equation (8) with the third rank tensor $\mathcal{A}_{sij}$ has disappeared due to the evident boundary condition $\mathcal{A}_{sij}(\mathbf{r}, t) \to 0$ at $\mathbf{r} \to \infty$, which follows from the boundary condition for the Wigner function $f(\mathbf{r}, \mathbf{p}, t) \to 0$ at $\mathbf{r} \to \infty$. As a result we are left with the closed system of equations for second rank tensors.

The equations (9) and (11) are evidently symmetrical with respect to indexes i, j and the equation (10) has not the definite symmetry. We can construct easily the symmetrical and antisymmetrical equations by combinations of the equation (10) with different indexes:

$$m \frac{d}{dt} [\int x_j n(\mathbf{r}, t) u_i(\mathbf{r}, t) d\mathbf{r}]_{ij} + [\int x_j n(\mathbf{r}, t) \frac{\partial V}{\partial x_i} d\mathbf{r}]_{ij} - \frac{2}{m} \Pi_{ij}(t) = 0, \tag{12}$$

$$m\frac{d}{dt}\int n(\mathbf{r},t)\{x_j u_i(\mathbf{r},t) - x_i u_j(\mathbf{r},t)\}d\mathbf{r} = -\int n(\mathbf{r},t)\left\{x_j\frac{\partial V}{\partial x_i} - x_i\frac{\partial V}{\partial x_j}\right\}d\mathbf{r}. \quad (13)$$

The integral on the left-hand side of the equation (13) is the angular momentum of the nucleus. When $V(\mathbf{r}, t)$ is a self-consistent potential, the right-hand side of this equation is equal to zero and the equation expresses the angular momentum conservation law. For our model potential

$$x_j\frac{\partial V}{\partial x_i} - x_i\frac{\partial V}{\partial x_j} = 2\lambda q(t)x_j x_i(\delta_{i1} - \delta_{j1} + \delta_{i2} - \delta_{j2} - 2\delta_{i3} + 2\delta_{j3}).$$

This expression is different from zero for $j = 3$, $i = 1, 2$ or $i = 3, j = 1, 2$ and proportional to x_1x_3 or x_2x_3. The corresponding integral $\int n(\mathbf{r}, t)x_1x_3d\mathbf{r}$ is equal to zero, because our potential does not destroy the triplanar symmetry of the nucleon distribution $n(\mathbf{r}, t)$. Therefore our model conserves the angular momentum.

The non-trivial information is contained in the symmetrical equation (12). We transform it using the equation (9) and the expression (5) for the potential derivative:

$$m\frac{d^2}{dt^2}J_{ij}(t) + 2J_{ij}(t)\{m\omega^2 + \lambda q(t)(\delta_{i1} + \delta_{j1} + \delta_{i2} + \delta_{j2} - 2\delta_{i3} - 2\delta_{j3})\} - \frac{2}{m}\Pi_{ij}(t) = 0. \quad (14)$$

By definition $q(t) = J_{11}(t) + J_{22}(t) - 2J_{33}(t)$, hence this equation is nonlinear in J_{ij}. As is seen from its structure we can write the set of coupled dynamical equations for the tensors $J_{11} + J_{22}$ and J_{33}:

$$m(\ddot{J}_{11} + \ddot{J}_{22}) + 2(J_{11} + J_{22})\{m\omega^2 + 2\lambda(J_{11} + J_{22} - 2J_{33})\} - \frac{2}{m}(\Pi_{11} + \Pi_{22}) = 0, \quad (15)$$

$$m\ddot{J}_{33} + 2J_{33}\{m\omega^2 - 4\lambda(J_{11} + J_{22} - 2J_{33})\} - \frac{2}{m}\Pi_{33} = 0, \quad (16)$$

where dot means the time derivative and we don't write out the time dependence of tensors for simplicity. To be closed, this system must be supplemented with the dynamical equations for the tensors $\Pi_{11} + \Pi_{22}$ and Π_{33}. They are easily obtained from the equation (11):

$$\frac{d}{dt}\Pi_{ii}(t) + 2m\{m\omega^2 + 2\lambda q(t)(\delta_{i1} + \delta_{i2} - 2\delta_{i3})\}\int n(\mathbf{r}, t)u_i(\mathbf{r}, t)x_i d\mathbf{r} = 0. \quad (17)$$

Using here equation (9) we have:

$$(\dot{\Pi}_{11} + \dot{\Pi}_{22}) + m(\dot{J}_{11} + \dot{J}_{22})\{m\omega^2 + 2\lambda(J_{11} + J_{22} - 2J_{33})\} = 0, \quad (18)$$

$$\dot{\Pi}_{33} + m\dot{J}_{33}\{m\omega^2 - 4\lambda(J_{11} + J_{22} - 2J_{33})\} = 0. \quad (19)$$

As one sees from the structure of the equations (15)–(19), it will be more convenient to rewrite them in terms of new variables: the component of the quadrupole moment

$Q_{20} = J_{11} + J_{22} - 2J_{33}$, the mean square radius $Q_{00} = J_{11} + J_{22} + J_{33}$ and the irreducible tensors $\Pi_{20} = \Pi_{11} + \Pi_{22} - 2\Pi_{33}$ and $\Pi_{00} = \sum_{s=1}^{3} \Pi_{ss}$. Taking the simple combinations of the equations (15), (16) and (18), (19) we get finally:

$$m^2\ddot{Q}_{00} + 2m^2\omega^2 Q_{00} + 4m\lambda Q_{20}^2 - 2\Pi_{00} = 0,$$

$$m^2\ddot{Q}_{20} + 2m^2\omega^2 Q_{20} + 4m\lambda Q_{20}(2Q_{00} - Q_{20}) - 2\Pi_{20} = 0,$$

$$\dot{\Pi}_{00} + m^2\omega^2\dot{Q}_{00} + 2m\lambda Q_{20}\dot{Q}_{20} = 0,$$

$$\dot{\Pi}_{20} + m^2\omega^2\dot{Q}_{20} + 2m\lambda Q_{20}(2\dot{Q}_{00} - \dot{Q}_{20}) = 0. \qquad (20)$$

Third equation of this system gives the integral of motion:

$$\Pi_{00} + m^2\omega^2 Q_{00} + m\lambda Q_{20}^2 = const. \qquad (21)$$

3. Analysis of the Equations of Motion

3.1. *Stationary Solution.* Investigating the stationary solution of the system (20) we can do some conclusion about the equilibrium shape of nuclei. By definition the variables of the stationary solution (or equilibrium state) don't depend on the time. Supposing the time derivatives in (20) equal to zero one gets two relations

$$m^2\omega^2 Q_{00} + 2m\lambda Q_{20}^2 - \Pi_{00} = 0,$$

$$m^2\omega^2 Q_{20} + 2m\lambda Q_{20}(2Q_{00} - Q_{20}) - \Pi_{20} = 0. \qquad (22)$$

We shall call them equations of equilibrium. The second relation is of a special importrance — it says that it is impossible to have a static quadrupole deformation ($Q_{20} \neq 0$) without a Fermi surface deformation ($\Pi_{20} \neq 0$) and vice versa [6]. Formally one can find non-trivial solution for Q_{20} having $\Pi_{20} = 0$:

$$Q_{20} = 2Q_{00} + \frac{m\omega^2}{2\lambda}. \qquad (23)$$

However it turns out, that for the self-consistent force constant [1, 7]

$$\lambda = \lambda_{\text{Bohr}} = \frac{-m\omega^2}{4A\langle r^2\rangle} \qquad (24)$$

the expression (23) is equal to zero (we remind that $Q_{00} = A\langle r^2\rangle$).

3.2. *Small Amplitude Approximation.* Let us consider the system (20) in the small amplitude approximation. Taking the variations $Q_{\lambda 0}(t) = Q_{\lambda 0}(0) + \delta Q_{\lambda 0}(t)$, $\Pi_{\lambda 0}(t) = \Pi_{\lambda 0}(0) + \delta\Pi_{\lambda 0}(t)$ and neglecting the terms quadratic in δ, one obtains two independent systems: the system for quadrupole tensors,

$$m^2\delta\ddot{Q}_{20} + (2m^2\omega^2 + 8m\lambda Q_{00}(0))\,\delta Q_{20} - 2\delta\Pi_{20} = 0,$$

$$\delta\dot{\Pi}_{20} + m^2\omega^2\delta\dot{Q}_{20} = 0, \tag{25}$$

and the system for monopole tensors,

$$m^2\delta\ddot{Q}_{00} + 2m^2\omega^2\delta Q_{00} - 2\delta\Pi_{00} = 0,$$

$$\delta\dot{\Pi}_{00} + m^2\omega^2\delta\dot{Q}_{00} = 0. \tag{26}$$

We consider the spherical nuclei in this paper, so we put everywhere $Q_{20}(0) = 0$. Supposing the time dependence $e^{i\Omega t}$ for all variables one can find easily the next eigenfrequencies:

$$\Omega_0 = 2\omega \tag{27}$$

for the monopole vibrations and

$$\Omega_2 = 2\sqrt{\omega^2 + \frac{2\lambda}{m}Q_{00}(0)} \tag{28}$$

for the quadrupole vibrations. Using in (28) the expression (24) for the force constant one obtains the well-known [1, 7] result for the quadrupole eigenfrequency

$$\Omega_2 = \sqrt{2}\,\omega\,. \tag{29}$$

The energies $E_0 = \hbar\Omega_0$ and $E_2 = \hbar\Omega_2$ are in qualitative agreement with experimental values of the energies of the monopole and quadrupole giant resonances (for $\hbar\omega = 41A^{-1/3}$ MeV).

So, in the small amplitude approximation our model gives only two levels, which can be interpreted as giant 0^+ and 2^+ resonances. This is true also for the calculations with realistic interactions [8].

3.3. *Numerical Solution and Fourier Analysis.* Principally another situation is observed in general case, when the system (20) is solved without any approximations. We solve it numerically with the help of Runge-Kutta procedure. The solutions depend strongly on the initial conditions (i.c.).

They can be chosen in two ways. In the first case one takes the equilibrium values (i.e. satisfying eqs. (22)) for the moments $Q_{\lambda 0}(0)$, $\Pi_{\lambda 0}(0)$ and some definite (nonzero) values for their derivatives $\dot{Q}_{\lambda 0}(0)$, $\dot{\Pi}_{\lambda 0}(0)$. From the physical point of view that means that one pushes the nucleus and forces it to deviate from the state of equilibrium. In another case one takes some nonequilibrium values for $Q_{\lambda 0}(0)$, $\Pi_{\lambda 0}(0)$ and zero values for their derivatives. In such a way one deviates the nucleus from the equilibrium «by hand» and it begins to vibrate due to the restoring force.

It is evident, that both variants are equivalent: for any set of the initial values of $Q_{\lambda 0}(0)$, $\Pi_{\lambda 0}(0)$, $\dot{Q}_{\lambda 0}(0)$, $\dot{\Pi}_{\lambda 0}(0)$ from the first variant one can always find the corresponding set in the second variant to get the equivalent final results.

In this paper the first variant is used. We take $Q_{20}(0) = \Pi_{20}(0) = 0$ because we deal with spherical nuclei. The evident approximation is used for the monopole momentum: $Q_{00}(0) = \frac{3}{5} R_0^2 A$, where $R_0 = 1.18A^{1/3}$. The initial value $\Pi_{00}(0)$ is fixed by the first equation of the system (22). The initial values for the time derivatives $\dot{Q}_{00}(0)$ and $\dot{Q}_{20}(0)$ are arbitrary. We have performed the calculations for two values of the force constant λ and for six sets of $\dot{Q}_{00}(0)$, $\dot{Q}_{20}(0)$: 1) 6100, 10000, 2) 5000, 15000, 3) 5000, 18000, 4) 6100, 10, 5) 10, 10000, 6) 10, 10 (all the values are in MeV · fm^2). The time-dependence of the function $Q_{20}(t)$ for the first variant of i.c. with $\lambda = \lambda_{\text{Bohr}}$ is demonstrated by the figure. As one can see, it oscillates quite irregularly. The maximal period of oscillations, when the curve begins to repeat itself, is $\tau_2 = 457.4$ MeV^{-1} ($\tau = t/\hbar$). The pictures for other functions and other variants of i.c. are more or less similar. Having the periods of oscillation one can perform the Fourier analysis of the curves and represent all the functions by series

$$f(t) = \frac{a_0}{2} + \sum_i (a_i \cos \omega_i t + b_i \sin \omega_i t).$$

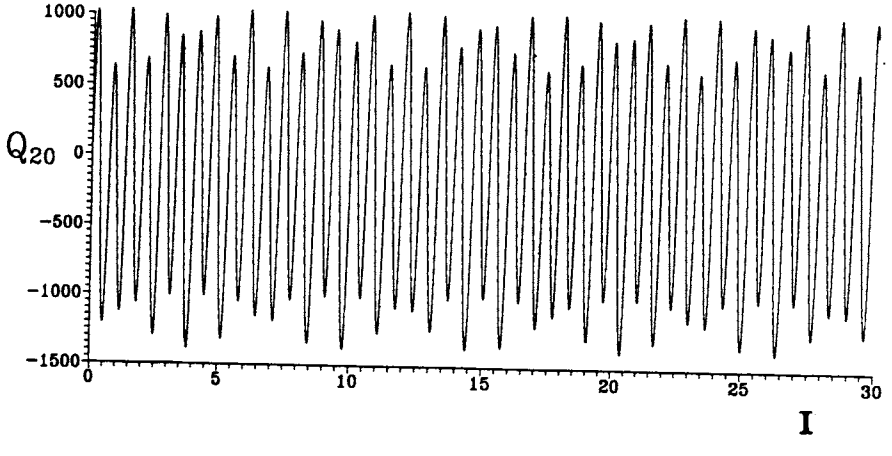

Fig. The time-dependence ($\tau = t/\hbar$) of the quadrupole moment for $\lambda = \lambda_{\text{Bohr}}$ and the initial conditions $\dot{Q}_{00}(0) = 6100$, $\dot{Q}_{20}(0) = 10^4$

Table 1. Fourier coefficients and energies for $\dot{Q}_{00}(0) = 6100$, $\dot{Q}_{20}(0) = 10^4$

i	$\hbar\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
0	0.00		0.000	69.547	0.000	-346.892
1	1.14	3Q-2M	-2.729	-0.735	9.464	2.550
2	1.90	5M-7Q	0.000	0.000	0.003	0.004
3	2.28	6Q-4M	0.006	-0.010	-0.038	0.065
4	3.04	3M-4Q	-0.086	0.035	0.407	-0.167
5	3.42	9Q-6M	0.000	0.000	-0.001	-0.002
6	4.18	M-Q	-0.641	-5.152	26.682	213.580
7	5.32	2Q-M	16.475	2.312	-46.538	-6.508
8	6.07	6M-8Q	0.000	0.000	0.001	0.001
9	6.46	5Q-3M	-0.005	0.012	0.015	-0.035
10	7.21	4M-5Q	0.029	-0.016	-0.172	0.097
11	8.35	2M-2Q	0.818	3.234	-8.459	-33.331
12	9.49	Q	-37.702	-0.585	1008.723	14.817
13	10.63	4Q-2M	0.342	-1.194	0.865	-3.034
14	11.39	5M-6Q	0.001	0.001	-0.029	0.021
15	11.77	7Q-4M	0.008	0.005	0.014	0.008
16	12.53	3M-3Q	-0.703	-1.803	-0.983	-2.512
17	13.67	M	461.418	-50.204	53.062	-5.838
18	14.81	3Q-M	2.606	-16.684	-0.640	4.139
19	15.95	6Q-3M	-0.020	-0.008	0.014	0.006
20	16.70	4M-4Q	-0.006	-0.011	0.029	0.053
21	17.84	2M-Q	1.833	-0.433	-7.259	1.729
22	18.98	2Q	-1.073	34.542	0.703	-23.925
23	20.12	5Q-2M	0.179	0.054	-0.128	-0.038
24	22.02	3M-2Q	0.238	-0.088	0.347	-0.130

i	$\hbar\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
25	23.16	Q+M	0.199	2.142	1.069	11.233
26	24.30	4Q-M	-0.140	-0.024	0.628	0.107
27	25.44	7Q-3M	0.000	0.000	0.001	-0.002
28	26.20	4M-3Q	-0.005	0.002	0.029	-0.015
29	27.34	2M	-0.041	-0.187	0.145	0.651
30	28.48	3Q	0.453	0.021	-1.055	-0.046
31	29.62	6Q-2M	0.001	-0.004	-0.003	0.009
32	31.51	3M-Q	0.001	+0.002	-0.012	-0.035
33	32.65	2Q+M	-0.130	0.010	0.191	-0.015
34	33.79	5Q-M	-0.001	0.009	0.003	-0.017
35	36.83	2M+Q	-0.015	0.000	-0.025	0.005
36	37.97	4Q	0.001	0.000	-0.002	0.027
37	41.00	3M	0.001	0.000	-0.003	0.001

Table 2. Fourier coefficients and energies for $\dot{Q}_{00}(0) = 5000$, $\dot{Q}_{20}(0) = 15000$

i	$\hbar\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
0	0.000		0.000	159.3703	0.0000	–629.2916
1	0.632	7M–10Q	–0.0016	–0.0003	0.0095	0.0026
2	1.155	3Q–2M	–6.2262	–6.8901	26.2050	29.0060
3	1.787	5M–7Q	–0.0027	0.0012	0.1114	–0.0543
4	2.310	6Q–4M	0.0315	0.0031	–0.5563	–0.0557
5	2.942	3M–4Q	–0.1478	0.3773	–0.1133	0.2919
6	3.466	9Q–6M	0.0043	–0.0034	–0.0185	0.0163
7	4.097	M–Q	–4.8411	–12.7965	108.3201	286.6512
8	4.621	12Q–8M	–0.0003	0.0003	–0.0001	–0.0034

i	$\hbar\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
9	5.253	2Q–M	32.5877	16.7307	–81.0800	–41.6762
10	5.884	6M–8Q	–0.0018	0.0025	–0.0098	0.0084
11	6.408	5Q–3M	0.3342	–0.0889	–1.0585	0.2804
12	7.040	4M–5Q	0.0040	–0.3266	–0.0159	1.2394
13	7.563	8Q–5M	–0.0116	0.0185	0.0522	–0.0833
14	8.195	2M–2Q	7.9911	9.0500	–56.5284	–64.1159
15	8.827	9M–12Q	0.0006	–0.0001	–0.0027	0.0020
16	9.350	Q	–96.8501	–10.9563	1496.3934	170.5650
17	9.982	7M–9Q	–0.0001	–0.0011	–0.0107	0.0273
18	10.505	4Q–2M	5.0277	–3.6061	18.3302	–13.1196
19	11.137	5M–6Q	–0.0197	–0.0539	0.0514	0.1395
20	11.660	7Q–4M	0.0037	–0.0177	–0.0295	0.1365
21	12.292	3M–3Q	–3.6049	–1.9033	–8.5572	–4.5308
22	12.815	10Q–6M	0.0111	0.0193	0.0057	0.0081
23	12.924	10M–13Q	–0.0004	0.0006	0.0008	0.0002
24	13.447	M	399.0070	–101.4652	102.3071	–25.8836
25	13.970	13Q–8M	0.0020	0.0014	–0.0008	0.0001
26	14.079	8M–10Q	–0.0006	–0.0044	–0.0005	0.0005
27	14.602	3Q–M	31.9651	–48.0564	–5.7075	8.5558
28	15.234	6M–7Q	0.0095	0.0097	0.0014	0.0020
29	15.757	6Q–3M	–0.1036	–0.6973	0.0584	0.3970
30	16.389	4M–4Q	–0.2989	–0.0378	0.2749	0.0349
31	16.913	9Q–5M	0.0110	0.0084	–0.0079	–0.0062
32	17.545	2M–Q	7.1178	–4.9824	11.3260	7.9007
33	18.700	2Q	–18.0739	78.8608	12.3700	–53.5567
34	19.332	7M–8Q	–0.0015	–0.0001	0.0006	0.0003

i	$h\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
35	19.855	5Q–2M	0.9875	1.7677	–0.6990	–1.2564
36	20.487	5M–5Q	–0.0634	0.0156	0.0196	–0.0047
37	21.010	8Q–4M	0.0044	0.0016	–0.0024	–0.0008
38	21.642	3M–2Q	0.6798	–0.9969	0.2118	–0.3093
39	22.165	11Q–6M	–0.0008	0.0004	–0.0001	0.0001
40	22.797	Q+M	0.8979	6.5434	1.6030	11.8635
41	23.952	4Q–M	–0.3578	–0.3010	2.4021	2.0305
42	24.584	6M–6Q	–0.0010	0.0007	–0.0020	0.0013
43	25.107	7Q–3M	–0.0283	0.0010	0.0851	–0.0028
44	25.739	4M–3Q	–0.0036	0.0150	0.0306	–0.1253
45	26.894	2M	–0.2367	–0.4352	0.7693	1.4230

Table 3. Fourier coefficients and energies for $\dot{Q}_{00}(0) = 5000, \dot{Q}_{20}(0) = 18000$

i	$h\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
0	0.000		0.000	248.004	0.000	–966.077
1	0.283	9M–13Q	–0.010	0.005	0.061	–0.033
2	0.952	3Q–2M	–8.682	–21.535	42.528	105.465
3	1.236	7M–10Q	0.001	0.000	0.084	–0.124
4	1.905	6Q–4M	–0.553	–0.574	0.865	0.897
5	2.189	5M–7Q	–0.062	0.229	0.310	–1.147
6	2.858	9Q–6M	0.116	0.052	–0.788	–0.357
7	3.141	3M–4Q	0.045	0.389	1.234	10.593
8	3.811	12Q–8M	0.010	0.001	–0.061	–0.015
9	4.094	M–Q	–12.323	–22.605	183.778	336.847
10	4.378	10M–14Q	–0.000	0.002	0.025	–0.020

i	$\hbar\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
11	4.764	15Q–10M	–0.001	0.002	0.014	–0.002
12	5.047	2Q–M	43.929	36.141	–87.113	–71.611
13	5.330	8M–11Q	–0.003	0.007	0.005	0.000
14	5.717	18Q–12M	–0.000	–0.001	0.001	0.003
15	6.000	5Q–3M	4.043	1.273	–14.550	–4.574
16	6.283	6M–8Q	–0.024	–0.100	0.162	0.671
17	6.953	8Q–5M	–0.201	0.015	1.112	–0.084
18	7.236	4M–5Q	–1.187	–1.679	3.013	4.258
19	7.906	11Q–7M	–0.032	0.012	0.132	–0.048
20	8.189	2M–2Q	33.679	21.708	–190.830	–122.828
21	8.472	11M–15Q	0.001	–0.001	–0.017	0.017
22	8.859	14Q–9M	–0.014	0.014	0.033	–0.058
23	9.142	Q	–170.626	–32.693	1783.589	340.434
24	9.425	9M–12Q	0.000	–0.007	0.021	0.031
25	10.095	4Q–2M	13.165	–2.585	102.969	–20.304
26	10.378	7M–9Q	–0.033	–0.037	0.257	0.294
27	11.048	7Q–4M	1.595	–1.038	2.371	–1.547
28	11.331	5M–6Q	–0.579	–0.284	–1.120	–0.547
29	12.000	10Q–6M	–0.112	0.161	–0.197	0.284
30	12.284	3M–3Q	–7.164	–0.526	–24.352	–1.766
31	12.953	13Q–8M	–0.024	0.090	–0.015	0.033
32	13.237	M	361.277	–115.654	154.458	–49.621
33	13.520	10M–13Q	–0.032	–0.039	–0.002	–0.009
34	14.189	3Q–M	112.204	–93.180	–3.016	2.511
35	14.473	8M–10Q	–0.003	0.006	0.007	+0.002
36	15.142	6Q–3M	4.595	–8.523	–1.928	3.586

i	$\hbar\omega_i$, MeV		Q_{00}		Q_{20}	
			b_i	a_i	b_i	a_i
37	15.426	6M–7Q	0.216	–0.006	0.034	–0.002
38	16.095	9Q–5M	0.005	–0.057	–0.004	0.047
39	16.378	4M–4Q	–3.315	1.504	1.591	–0.724
40	17.331	2M–Q	23.410	–24.538	–21.116	22.195
41	18.001	15Q–9M	0.006	0.014	–0.006	–0.005
42	18.284	2Q	–46.772	117.573	30.189	–76.202
43	19.237	5Q–2M	0.056	12.158	–0.052	–8.678
44	19.520	7M–8Q	–0.054	0.031	0.024	–0.014
45	20.190	8Q–4M	0.237	0.580	–0.167	–0.407

The results of such calculations are demonstrated in Tables 1, 2, 3, where the eigenfrequencies $\hbar\omega_i$ and the corresponding coefficients a_i and b_i of the functions Q_{20} and Q_{00} are shown for three variants of i.c. and $\lambda = \lambda_{\text{Bohr}}$. Let us analyse in detail the first table. As one can see there are about 40 eigenfrequencies having the diapason of the amplitudes a_i, b_i from 10^{-3} to 10^3, the dozen of them having this diapason from 10^0 to 10^3. All these frequencies correspond to transitions between various levels E_ν of the nucleus, i.e., they can be represented as differences $\hbar\omega_{\mu\nu} = E_\mu - E_\nu$. So, it is necessary to perform some combinatorial analysis to find the eigenvalues E_ν. Of course the energies of GQR and GMR can be recognized immediately without any combinatorics. They are very close to that of calculated in the small amplitude approximation: $E_2(\hbar\omega_{12})$ became 9.49 MeV instead of 9.78 MeV and $E_0(\hbar\omega_{17})$ became 13.67 MeV instead of 13.84 MeV. So, we confirmed the well-known fact, that giant resonances are described very well in the small amplitude approximation.

It is very interesting to discover the multiphonon states. One can find two- three- and four-phonon states, corresponding to GQR. Their energies are $\hbar\omega_{22} = 2 \cdot E_2 = 18.98$ MeV, $\hbar\omega_{30} = 3 \cdot E_2 = 28.48$ MeV and $\hbar\omega_{36} = 4 \cdot E_2 = 37.97$ MeV. There are two- and three-phonon states corresponding to GMR. Their energies are $\hbar\omega_{29} = 2 \cdot E_0 = 27.34$ MeV and $\hbar\omega_{37} = 3 \cdot E_0 = 41$ MeV. There is one two-phonon state consisting of the quadrupole and monopole phonons (its energy is $\hbar\omega_{25} = E_2 + E_0 = 23.16$ MeV). There are two three-phonon states consisting of: two quadrupole plus one monopole phonons $\hbar\omega_{33} = 2 \cdot E_2 + E_0 = 32.65$ MeV and two monopole plus one quadrupole phonons $\hbar\omega_{35} = E_2 + 2 \cdot E_0 = 36.83$ MeV.

It is not so difficult to show, that all the rest $h\omega_i$ are just the differences of these (and more high lying) multiphonon states. The results of the combinatorial analysis are shown in the third columns of the tables.

The calculations with the $\lambda = \frac{3}{4}\lambda_{\text{Bohr}}$ show that the results are rather sensitive to the force constant. For example, the energies of GQR and GMR are increased by 1.2 MeV and 0.06 MeV, respectively, their strengths decreasing about 15%. The strengths of multiphonon states are decreased about 2 times and more. The comparison of the results of calculations with different i.c. shows that strengths of all the states are very sensitive to i.c., what is evident. Not so evident is the noticeable dependence of energies on i.c. We interpret it as the manifestation of the dynamical deformation of the nucleus. This deformation is rather large and depends on i.c. Analysing tables 1–3 one can notice the next rule: the more (the less) the initial values of Q_{00} or Q_{20} are chosen, the more (the less) the resulting amplitudes are obtained and the more of new frequencies appear.

The limit of maximum possible amplitudes is achieved at $Q_{20}(0) \cong 20000$. The calculations show that the maximum positive value of Q_{20} is ~1980 and the maximum negative value is ~2480. Which value of the deformation parameter β these amplitudes correspond to? To answer this question we derive the formula for the β-dependence of Q_{20} in the approximation of the sharp edge of a nucleus. By definition

$$Q_{20}(\beta) = n_0(\beta) \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{R(\theta,\phi)} (x_1^2 + x_2^2 - 2x_3^2) r^2 dr. \tag{30}$$

Here $R(\theta, \phi) = R_0(1 + \beta Y_{20}(\theta,\phi))$, $x_1^2 + x_2^2 - 2x_3^2 = -4\sqrt{\frac{\pi}{5}}\, r^2 Y_{20}(\theta,\phi)$ and the density $n_0(\beta)$ is defined as

$$n_0(\beta) = A \left\{ \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \int_0^{R(\theta,\phi)} r^2 dr \right\}^{-1}.$$

Performing the simple but tedious calculations we get:

$$Q_{20}(\beta) = -4\sqrt{\frac{\pi}{5}} n_0(\beta) R_0^5 \left\{ \beta + \frac{4}{7}\sqrt{\frac{5}{4\pi}}\beta^2 + \frac{15}{14\pi}\beta^3 + \right.$$

$$\left. + \frac{100}{77}\sqrt{\frac{5}{(4\pi)^3}}\beta^4 + \frac{25 \cdot 53}{77 \cdot 13(4\pi)^2}\beta^5 \right\},$$

$$n_0(\beta) = 3\frac{A}{R_0^3}\left\{ 4\pi + 3\beta^2 + \frac{2}{7}\sqrt{\frac{5}{4\pi}}\beta^3 \right\}^{-1}. \tag{31}$$

With the help of these formulae we find, that the vibrations with the maximum amplitude correspond to the change of β from ~0.29 to ~–0.29. So, the shape of the nucleus changes during the vibrations from oblate to prolate. Further increasing of $\dot{Q}_{20}(0)$ leads to the instability: the amplitudes begin to grow infinitely. It is necessary to stress that due to the lack of the full self-consistency of our Hamiltonian this value of maximum β must be considered as the lower bound for β_{max}. By the way, the amplitudes presented on the figure correspond to $\beta \cong 0.13$.

The limit of small amplitudes is practically achieved for the sixth variant of i.c., where only GQR and GMR have the noticeable amplitudes and their energies are equal exactly to $\sqrt{2}\hbar\omega$ and $2\hbar\omega$.

3.4. *Hydrodynamical Limit.* There is one more interesting solution of the system (20). It is the so-called «hydrodynamical limit». The Fermi liquid differs from the classic one by the existence of the Fermi surface deformation. So, if to suppose $\Pi_{20} = 0$, the system (20) (without the last equation) will describe pure hydrodynamics. Its solution in the small amplitude approximation is very well known: $E_2 = 0$, i.e., there is no GQR in the classic hydrodynamics. However the exact solution gives the nonzero value: $E_2 = 0.28$ MeV for $\dot{Q}_{20}(0) = 3$, $\dot{Q}_{00}(0) = 6100$. This solution exists for initial conditions $\dot{Q}_{20}(0) < 3.5$, $\dot{Q}_{00}(0) > 6000$ and changes not very much in these limits. Including Π_{20} gradually one can observe the evolution of this solution. With this aim we multiplied Π_{20} in the third equation of the system (20) by a constant factor α. When α is changed gradually from 0 to 1, E_2 grows gradually from 0.28 MeV at $\alpha = 0$ to the usual value of the GQR energy at $\alpha = 1$.

3.5. *Excitations Probabilities.* The excitations probabilities can be calculated with the help of the classical formula for the intensity of the quadrupole radiation [9]:

$$\mathrm{Int} = \frac{1}{180c^5} \sum_{i,j} \bar{D}_{i,j}^2 , \tag{32}$$

where $D_{i,j} = eZ/A\left(3J_{i,j} - \delta_{i,j}\sum_s J_{ss}\right)$. Due to the axial symmetry of the Hamiltonian $D_{11} = D_{22}$, $D_{33} = -2D_{11}$ and $D_{i,j} = 0$ for $i \neq j$. Hence $\sum_{i,j} D_{i,j}^2 = \frac{3}{2}D_{33}^2 = \frac{3}{2}\left(\frac{eZ}{A}\right)^2 Q_{20}^2$. Putting into (32) the Fourier expansion for Q_{20} and averaging over the greatest period of oscillations we get:

$$\overline{\mathrm{Int}} = \left(\frac{eZ}{A}\right)^2 \frac{1}{120c^5} \sum_\alpha \omega_\alpha^6 \frac{a_\alpha^2 + b_\alpha^2}{2} \equiv \sum_\alpha \overline{\mathrm{Int}}_\alpha . \tag{33}$$

Dividing $\overline{\mathrm{Int}}_\alpha$ by $\hbar\omega_\alpha$ we obtain the radiation probability W_α. Taking into account the relation between W_α and the reduced probability [10] we find:

$$B_\alpha(E2) = \left(\frac{eZ}{A}\right)^2 \frac{5}{64\pi}(a_\alpha^2 + b_\alpha^2) = \left(\frac{Z}{A}\right)^2 \frac{125}{144}\frac{a_\alpha^2 + b_\alpha^2}{R^4} B_W, \tag{34}$$

where B_W is Weisskopf unit.

Using here the values of a_α and b_α from Table 1 we can calculate the $B(E2)$-factors for GQR and multiphonon states:

$$B(E2, \text{GQR}) = 57.5 B_W,$$

$$B(E2, 2 \times \text{GQR}) \equiv 6 \cdot 10^{-4} B(E2, \text{GQR}) = 0.03 B_W .$$

The excitation probability of the two-phonon GQR is approximately three orders of magnitude less than that of the usual one-phonon GQR. The $B(E2)$-factor for the three-phonon state is six orders of magnitude less than that of the GQR.

4. One-Dimensional Model

To reach more deep understanding of rather unusual properties of our model (the dependence of eigenfrequences on initial conditions, the lack of an anharmonicity of a spectrum in spute of an anharmonic potential) we will consider here exactly soluble one-dimensional model of a harmonic oscillator with a monopole-monopole residual interaction. Its solution was found by Reinhardt and Schulz [11] in a rather complicated way. With the help of our method the solution becomes elementary.

The average field of the model (in the notations of [11]) is

$$V(x, t) = \frac{1}{2} m\omega_0^2 x^2 = \kappa(< x^2 > - x_0^2)(x^2 - x_0^2/A), \tag{35}$$

where in correspondence with our notations $\omega_0 = \omega$, $< x^2 > = J_{11}(t)$, $x_0^2 = J_{11}(0)$. Following the rules described in section 2.1 one can derive the system of equations

$$m\ddot{J} + 2J[m\omega^2 + 2\kappa(J - J_0)] - \frac{2}{m}\Pi = 0,$$

$$\dot{\Pi} + m\dot{J}[m\omega^2 + 2\kappa(J - J_0)] = 0 \tag{36}$$

with $J = J_{11}(t)$, $J_0 = J_{11}(0)$, $\Pi = \Pi_{11}$. The second equation of this system gives the integral of motion

$$\Pi + m^2\omega^2 J + m\kappa J^2 - 2m\kappa J_0 J = \text{const}. \tag{37}$$

The value of const can be fixed by the conditions of equilibrium. In the state of equilibrium $J = J_0$, $\Pi = \Pi_0$ and one has from (36) and (37):

$$2m\omega^2 J_0 - \frac{2}{m}\Pi_0 = 0,$$

$$\Pi_0 + m^2\omega^2 J_0 - m\kappa J_0^2 = \text{const}. \tag{38}$$

Combining these two equations one finds:

$$\text{const} = 2m^2\omega^2 J_0^2 - m\kappa J_0^2. \tag{39}$$

Using (37), (39) and introducing new variable $y = J - J_0$ one reduces the system (36) to the single equation

$$\ddot{y} + ay + by^2 = 0, \tag{40}$$

with $a = 4\left(\omega^2 + \frac{\kappa}{m}J_0\right)$, $b = 6\frac{\kappa}{m}$. This equation is integrated trivially to give

$$\left(\frac{dy}{dt}\right)^2 = -\frac{2}{3}by^3 - ay^2 - c_1, \tag{41}$$

where c_1 is a constant of integration, which is determined by initial conditions. Having in mind, that $y(0) = 0$, one finds $c_1 = -(\dot{y}(0))^2$. The solution of the equation (41) can be expressed in terms of the Jacobian elliptic function [12]:

$$y(t) = \eta_3 + (\eta_2 - \eta_3)\,\mathrm{sn}^2(\bar{\omega}t). \tag{42}$$

Here $\bar{\omega} = \frac{\omega}{x_o}\sqrt{\bar{\kappa}(\eta_1 - \eta_3)}$, $\bar{\kappa} = \kappa\frac{x_0^2}{m\omega^2}$, η_i are the roots of the polynomial

$$P(y) = y^3 + \frac{3a}{2b}y^2 + \frac{3c_1}{2b}. \tag{43}$$

The function $\mathrm{sn}(\phi)$ is a periodical one with a period $\Delta\phi = 4\mathbf{K}$, $\mathbf{K}$ being the complete elliptic integral of a first kind:

$$\mathbf{K} = \int_0^{\pi/2}\frac{d\phi}{\sqrt{1 - k^2\sin\phi}}, \tag{44}$$

where $k^2 = \frac{\eta_2 - \eta_3}{\eta_1 - \eta_3}$. Hence, the period of the function $y(t)$ will be proportional to $\Delta t = \frac{4\mathbf{K}}{\omega}$ and the corresponding frequency will be proportional to $\Omega = \frac{2\pi}{\Delta t} = \frac{\pi\bar{\omega}}{2\mathbf{K}}$. This expression demonstrates very well the dependence of eigenfrequencies on initial conditions,

because it is evident that the roots of the polynomial (43) depend on c_1, which in its turn depends on $\dot{y}(0)$. The absence of any anharmonicity can be understood studying the trigonometrical expansion of the Jacobian function [13]:

$$\mathrm{sn}\bar{\omega}t = \frac{2\pi}{k\mathbf{K}} \sum_{n=1}^{\infty} \frac{q^{n-1/2}}{1-q^{2n-1}} \sin(2n-1)\frac{\bar{\omega}\pi}{2\mathbf{K}}t.$$

Here $q = \exp(-\pi\mathbf{K}'/\mathbf{K})$, $\mathbf{K}' = \mathbf{K}(k')$, $k' = \sqrt{1-k^2}\mathrm{B}$ This expansion contains only frequencies proportional to odd numbers of the basic frequency $\Omega = \frac{\bar{\omega}\pi}{2\mathbf{K}}$. It is evident, that sn^2 will contain frequencies $n\Omega$ with even n only. So, the Fourier expansion of the function $y(t)$ will contain only one basic frequency 2Ω and its satellites 4Ω, 6Ω and so on. In the case of our two-dimensional problem of a coupled dynamics of monopole and quadrupole moments it is natural to expect the two basic frequencies Ω_1, Ω_2 and their satellites. Due to coupling there must be a lot of linear combinations of these frequencies: $n_1\Omega_1 \pm n_2\Omega_2$. As we have seen, such picture really takes place.

Let us compare our solution with that of Reinhardt and Schulz [11]. They have studied the collective variable $r(t)$ which is connected with our variable $y(t)$ by the relation $y = x_0^2(r^2-1)$ (formula (3.28) of [11]). Their dynamical equation for $r(t)$ reads (formulae (3.17), (3.29)):

$$\ddot{r} - \frac{\omega^2}{r^3} + \omega^2[r + 2\bar{\kappa}(r^3 - r)] = 0. \tag{45}$$

Multiplying (45) by $\dot{r}$ one easily transforms it into

$$\frac{d}{dt}\left\{\frac{\dot{r}^2}{2} + \frac{\omega^2}{2r^2} + \omega^2\left[\frac{r^2}{2} + \bar{\kappa}\left(\frac{r^4}{2} - r^2\right)\right]\right\} = 0, \tag{46}$$

demonstrating the existence of the integral of motion

$$\dot{r}^2 + \omega^2\left[\frac{1}{r^2} + r^2 + \bar{\kappa}(r^2-1)^2\right] = c_2, \tag{47}$$

which expresses the energy conservation. This integral allows one to prove the equivalence of the equations (40) and (45). Really, putting $y = x_0^2(r^2-1)$ and $\ddot{y} = 2x_0^2(r\ddot{r} + \dot{r}^2)$ into (40) and eliminating the term proportional to $\dot{r}^2$ with the help of the relation (47) one gets:

$$r\ddot{r} - \omega^2\left[\frac{1}{r^2} + r^2 + \bar{\kappa}(r^2-1)^2\right] + c_2 + \frac{a}{2}(r^2-1) + \frac{b}{2}x_0^2(r^2-1)^2 = 0. \tag{48}$$

This equation becomes equivalent to equation (45) if to take $c_2 = 2\omega^2$. With such value of c_2 our integral of motion (47) will coincide with that of Reinhardt and Schulz only in the case when $E_{HF} = E_0$ (formula (3.30)). By the way, this requirement follows naturally from their condition of self-consistence (see the bottom of section 3.2 in their paper [11]).

5. Conclusion

Let us enumerate the main results of this paper. The set of nonlinear dynamical equations for quadrupole Q_{20} and monopole Q_{00} moments of nuclei is derived from the TDHF equation with the help of the method of Wigner function moments. Due to the simplicity of the used Hamiltonian all the derivations are performed exactly, without any approximations. These equations are solved numerically for ^{208}Pb. It is found, that the functions $Q_{20}(t)$ and $Q_{00}(t)$ oscillate irregularly. Their Fourier analysis yields a lot of eigenfrequencies, which correspond to various differences of the energy levels. Combinatorial analysis allows one to find the giant quadrupole and monopole resonances and several multiphonon states constructed of these two resonances. It is shown that the reduced probability of the excitation of the two-phonon giant quadrupole resonance is three orders of magnitude less than that of the one-phonon GQR.

The theory can be modified to take into account spin degrees of freedom. In this case it will be possible to study a large amplitude motion with the rather realistic Nilsson potential. The extension to the description of excitations of higher multipolarities is straightforward.

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